

Kolumban Hutter · Yongqi Wang ·  
Irina P. Chubarenko

# Physics of Lakes

Volume 1: Foundation of the Mathematical  
and Physical Background



 Springer

# Contents

<b>1 Introduction</b> .....	1
1.1 Motivation .....	1
1.2 Lakes on Earth .....	10
1.3 Lakes Characterised by Their Response to the Driving Environment .....	14
1.3.1 Seasonal Characteristics .....	14
1.3.2 Characteristics by Mixing .....	15
1.3.3 Boundary-Related Processes .....	18
1.3.4 Characterisation by Typical Scales .....	20
References .....	22
<b>2 Mathematical Prerequisites</b> .....	25
2.1 Scalars and Vectors .....	26
2.2 Tensors .....	38
2.3 Fields and Their Differentiation .....	41
2.4 Gradient, Divergence and Rotation of Vector and Tensor Fields ...	50
2.5 Integral Theorems of Vector Analysis .....	60
2.5.1 GAUSS Theorems .....	60
2.5.2 STOKES Theorems .....	62
References .....	65
<b>3 A Brief Review of the Basic Thermomechanical</b> .....	67
<b>Laws of Classical Physics</b> .....	67
3.1 Underlying Fundamentals – General Balance Laws .....	67
3.2 Physical Balance Laws .....	73
3.2.1 Balance of Mass .....	73
3.2.2 Balance of Linear Momentum .....	74
3.2.3 Balance of Moment of Momentum .....	76
3.2.4 Balance of Energy .....	77
3.2.5 Second Law of Thermodynamics .....	79
References .....	82

<b>4</b>	<b>Fundamental Equations of Lake Hydrodynamics</b>	83
4.1	Kinematics	84
4.2	Balance of Mass	100
4.3	Balances of Momentum and Moment of Momentum, Concept of Stress, Hydrostatics	110
4.3.1	Stress Tensor	113
4.3.2	Local Balance Law of Momentum or Newton's Second Law	118
4.3.3	Material Behaviour	123
4.3.4	Hydrostatics	128
4.4	Balance of Energy: First Law of Thermodynamics	136
4.5	Diffusion of Suspended Substances	141
4.6	Summary of Equations	146
4.7	A First Look at the Boussinesq and Shallow-Water Equations	150
	References	155
<b>5</b>	<b>Conservation of Angular Momentum–Vorticity</b>	157
5.1	Circulation	157
5.2	Simple Vorticity Theorems	167
5.3	Helmholtz Vorticity Theorem	170
5.4	Potential Vorticity Theorem	177
	References	184
<b>6</b>	<b>Turbulence Modelling</b>	185
6.1	A Primer on Turbulent Motions	185
6.1.1	Averages and Fluctuations	185
6.1.2	Filters	187
6.1.3	Isotropic Turbulence	190
6.1.4	REYNOLDS Versus FAVRE Averages	192
6.2	Balance Equations for the Averaged Fields	194
6.2.1	Motivation	194
6.2.2	Averaging Procedure	195
6.2.3	Averaged Density Field $\langle \rho \rangle$	197
6.2.4	Dissipation Rate Density $\langle \phi \rangle$	198
6.2.5	Reynolds Stress Hypothesis	198
6.2.6	One- and Two-Equation Models	201
6.3	$k$ - $\epsilon$ Model for Density-Preserving and Boussinesq Fluids	203
6.3.1	The Balance Equations	203
6.3.2	Closure Relations	204
6.3.3	Summary of $(k - \epsilon)$ -Equations	206
6.3.4	Boundary Conditions	207
6.4	Final Remarks	210
6.4.1	Higher Order RANS Models	210

6.4.2	Large Eddy Simulation and Direct Numerical Simulation	211
6.4.3	Early Anisotropic Closure Schemes	212
	References	219
<b>7</b>	<b>Introduction to Linear Waves</b>	221
7.1	The Linear Wave Equation and Its Properties	222
7.2	Surface Gravity Waves Without Rotation	234
7.2.1	Short-Wave Approximation	245
7.2.2	Long-Wave Approximation	246
7.2.3	Standing Waves – Reflection	247
7.3	Free Linear Oscillations in Rectangular Basins of Constant Depth	252
7.4	Concluding Remarks	258
	References	261
<b>8</b>	<b>The Role of the Distribution of Mass Within Water Bodies on Earth</b>	263
8.1	Motivation	263
8.2	Processes of Surface Water Penetration to Depth	268
8.3	Homogenisation of Water Masses Requires Energy	274
8.3.1	Constant Density Layers	275
8.3.2	Continuous Density Variation	280
8.3.3	Influence of the Thermal Expansion	283
8.4	Motion of Buoyant Bodies in a Stratified Still Lake	285
8.4.1	Influence of Friction	290
8.5	Internal Oscillations – The Dynamical Imprint of the Density Structure	294
8.5.1	Fundamental Equations	297
8.5.2	Eigenvalue Problem for the Vertical Mode Structure in Constant Depth Basins	301
8.6	Closure	315
	References	317
<b>9</b>	<b>Vertical Structure of Wind-Induced Currents in Homogeneous and Stratified Waters</b>	319
9.1	Preview and Scope of This Chapter	319
9.2	Hydrodynamic Equations Applied to a Narrow Lake Under Steady Wind	322
9.2.1	Wind-Induced Steady Circulation in a Narrow Homogeneous Lake of Constant Depth	322
9.2.2	Influence of Bottom Slip on the Wind-Induced Circulation	328
9.2.3	Wind-Induced Steady Circulation in a Narrow Lake Stratified in Two Layers	330

9.3	Ekman Theory and Some of Its Extensions .....	340
9.3.1	Ekman Spiral .....	341
9.3.2	Steady Wind-Induced Circulation in a Homogeneous Lake on the Rotating Earth .....	358
9.3.3	Wind-Driven Steady Currents in Lake Erie .....	364
9.3.4	Time-Dependent Wind-Induced Currents in Shallow Lakes on the Rotating Earth .....	369
9.3.5	The Dynamical Prediction of Wind Tides on Lake Erie ...	376
9.4	Final Remarks .....	384
	References .....	385
<b>10</b>	<b>Phenomenological Coefficients of Water .....</b>	<b>389</b>
10.1	Density of Water .....	390
10.1.1	Natural Water and Sea Water .....	393
10.1.2	Suspended Matter .....	398
10.2	Specific Heat of Water .....	399
10.2.1	Specific Heat of Salty Water .....	399
10.3	Viscosity of Water .....	404
10.3.1	Pure Water .....	405
10.3.2	Sea Water .....	406
10.3.3	Natural Water .....	409
10.3.4	Suspended Matter .....	410
10.4	Molecular Heat Conductivity of Water .....	412
10.4.1	Heat Conductivity of Salt Water .....	413
10.4.2	Impurities .....	414
	References .....	416
	<b>Name Index .....</b>	<b>419</b>
	<b>Lake Index .....</b>	<b>423</b>
	<b>Subject Index .....</b>	<b>425</b>